X(10th) **Problems involving heights and distances**

A kite is flying with a string of length 200 m. If the thread makes an angle 30° with the ground, find the distance of the kite from the ground level. (Here, assume that the string is along a straight line)

Solution Let h denote the distance of the kite from the ground level.

In the figure, AC is the string

Given that $\angle CAB = 30°$ and $AC = 200$ m

In the right $\triangle CAB$, $\sin 30° = \frac{h}{200}$

$\implies$ $h = 200 \sin 30°$

$\therefore$ $h = 200 \times \frac{1}{2} = 100$ m

Fig. 7.7

Hence, the distance of the kite from the ground level is 100 m.

Find the angular elevation (angle of elevation from the ground level) of the Sun when the length of the shadow of a 30 m long pole is $10\sqrt{3}$ m.

Solution Let S be the position of the Sun and BC be the pole.

Let AB denote the length of the shadow of the pole.

Let the angular elevation of the Sun be θ.

Given that $AB = 10\sqrt{3}$ m and

$BC = 30$ m

In the right $\triangle CAB$, $\tan\theta = \frac{BC}{AB} = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}}$

$\implies$ $\tan\theta = \sqrt{3}$

$\therefore$ $\theta = 60°$

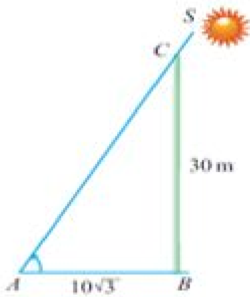


Fig. 7.9

Thus, the angular elevation of the Sun from the ground level is 60°.

The angle of elevation of the top of a tower as seen by an observer is 30°. The observer is at a distance of $30\sqrt{3}$ m from the tower. If the eye level of the observer is 1.5 m above the ground level, then find the height of the tower.

Solution Let BD be the height of the tower and AE be the distance of the eye level of the observer from the ground level.

Draw EC parallel to AB such that $AB = EC$.

Given $AB = EC = 30\sqrt{3}$ m and

$AE = BC = 1.5$m

In right angled $\triangle DEC$,

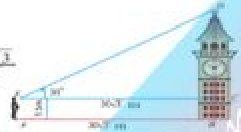
$$\tan 30^\circ = \frac{CD}{EC}$$

$$\Longrightarrow \quad CD = EC \tan 30^\circ = \frac{30\sqrt{3}}{\sqrt{3}}$$

$$\therefore \quad CD = 30\,\text{m}$$

Thus, the height of the tower, $BD = BC + CD$

$$= 1.5 + 30 = 31.5\,\text{m}$$



A vertical tree is broken by the wind. The top of the tree touches the ground and makes an angle 30 with it. If the top of the tree touches the ground 30 m away from its foot, then find the actual height of the tree. Ans: 30√3

A jet fighter at a height of 3000 m from the ground, passes directly over another jet fighter at an instance when their angles of elevation from the same observation point are 60° and 45° respectively. Find the distance of the first jet fighter from the second jet at that instant. (use $\sqrt{3} = 1.732$)

Let A and B be the positions of the two jet fighters at the given instant when one is directly above the other.

Let C be the point on the ground such that $AC = 3000$ m.

Given $\angle AOC = 60^\circ$ and $\angle BOC = 45^\circ$

Let h denote the distance between the jets at the instant.

In the right angled $\triangle BOC$, $\tan 45^\circ = \frac{BC}{OC}$

$$\Longrightarrow \quad OC = BC \qquad (\because \tan 45^\circ = 1)$$

Thus, $OC = 3000 - h$ (1)

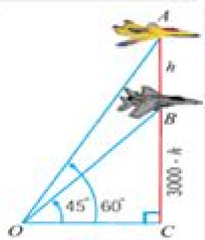


Fig. 7.12

In the right angled $\triangle AOC$, $\tan 60^\circ = \frac{AC}{OC}$

$$\Longrightarrow \quad OC = \frac{AC}{\tan 60^\circ} = \frac{3000}{\sqrt{3}}$$

$$= \frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 1000\sqrt{3} \qquad (2)$$

From (1) and (2), we get $3000 - h = 1000\sqrt{3}$

$$\Longrightarrow \quad h = 3000 - 1000 \times 1.732 = 1268\,\text{m}$$

The distance of the first jet fighter from the second jet at that instant is 1268 m.

The angle of elevation of the top of a hill from the foot of a tower is 60 and the angle of elevation of the top of the tower from the foot of the hill is 30. If the tower is 50 m high, then find the height of the hill

The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50m high, then find the height of the hill.

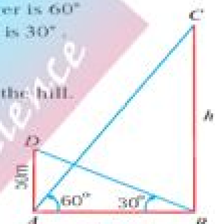


Fig. 7.13

Solution Let AD be the height of tower and BC be the height of the hill.

Given $\angle CAB = 60°$, $\angle ABD = 30°$ and $AD = 50$m.

Let BC = h metres.

Now, in the right angled ΔDAB, $\tan 30° = \frac{AD}{AB}$

$$\Longrightarrow \quad AB = \frac{AD}{\tan 30°}$$

$$\therefore \quad AB = 50\sqrt{3}\text{ m} \qquad (1)$$

Also, in the right angled ΔCAB, $\tan 60° = \frac{BC}{AB}$

$$\Longrightarrow BC = AB \tan 60°$$

Thus, using (1) we get $h = BC = (50\sqrt{3})\sqrt{3} = 150$ m

Hence, the height of the hill is 150 m.

A vertical wall and a tower are on the ground. As seen from the top of the tower, the angles of depression of the top and bottom of the wall are 45° and 60° respectively. Find the height of the wall if the height of the tower is 90 m. (use $\sqrt{3} = 1.732$)

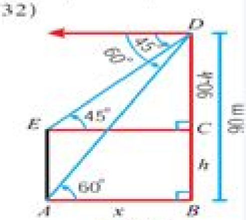


Fig. 7.14

Solution Let AE denote the wall and BD denote the tower.

Draw EC parallel to AB such that $AB = EC$. Thus, $AE = BC$.

Let $AB = x$ metres and $AE = h$ metres.

Given that $BD = 90$ m and $\angle DAB = 60°$, $\angle DEC = 45°$.

Now, $AE = BC = h$ metres

Thus, $CD = BD - BC = 90 - h$.

In the right angled ΔDAB, $\tan 60° = \frac{BD}{AB} = \frac{90}{x}$

$$\Longrightarrow \quad x = \frac{90}{\sqrt{3}} = 30\sqrt{3} \qquad (1)$$

In the right angled ΔDEC, $\tan 45° = \frac{DC}{EC} = \frac{90-h}{x}$

$$\text{Thus,} \quad x = 90 - h \qquad (2)$$

From (1) and (2), we have $90 - h = 30\sqrt{3}$

Thus, the height of the wall, $h = 90 - 30\sqrt{3} = 38.04$ m.

A girl standing on a lighthouse built on a cliff near the seashore, observes two boats due East of the lighthouse. The angles of depression of the two boats are 30° and 60°. The distance between the boats is 300 m. Find the distance of the top of the lighthouse from the sea level.

Solution Let A and D denote the foot of the cliff and the top of the lighthouse respectively.

Let B and C denote the two boats.

Let h metres be the distance of the top of the lighthouse from the sea level.

Let $AB = x$ metres.

Given that $\angle ABD = 60°$, $\angle ACD = 30°$

In the right angled ΔABD,

$$\tan 60° = \frac{AD}{AB}$$

$$\Longrightarrow \quad AB = \frac{AD}{\tan 60°}$$

Thus, $$x = \frac{h}{\sqrt{3}}$$

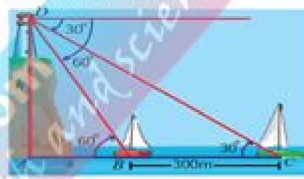


Fig. 7.15

Also, in the right angled ΔACD, we have

$$\tan 30° = \frac{AD}{AC}$$

$$\Longrightarrow \quad AC = \frac{AD}{\tan 30°} \Longrightarrow x + 300 = \frac{h}{\left(\frac{1}{\sqrt{3}}\right)}$$

Thus, $$x + 300 = h\sqrt{3}$$ (2

Using (1) in (2), we get $\frac{h}{\sqrt{3}} + 300 = h\sqrt{3}$

$$\Longrightarrow h\sqrt{3} - \frac{h}{\sqrt{3}} = 300$$

$\therefore$ $2h = 300\sqrt{3}$. Thus, $h = 150\sqrt{3}$.

Hence, the height of the lighthouse from the sea level is $150\sqrt{3}$ m.

A boy spots a balloon moving with the wind in a horizontal line at a height of 88.2 r from the ground level. The distance of his eye level from the ground is 1.2 m. The angle c elevation of the balloon from his eyes at an instant is 60°. After some time, from the sam point of observation, the angle of elevation of the balloon reduces to 30°. Find the distanc covered by the balloon during the interval.

Solution Let A be the point of observation.

Let E and D be the positions of the balloon when its angles of elevation are 60° and 30° respectively.

Let B and C be the points on the horizontal line such that $BE = CD$.

Let A', B' and C' be the points on the ground such that

$$A'A = B'B = C'C = 1.2 \text{ m}.$$

Given that $\angle EAB = 60^\circ$, $\angle DAC = 30^\circ$

$$BB' = CC' = 1.2 \text{ m and } C'D = 88.2 \text{ m}.$$

Also, we have $BE = CD = 87$ m.

Now, in the right angled $\triangle EAB$, we have

$$\tan 60^\circ = \frac{BE}{AB}$$

Thus, $$AB = \frac{87}{\tan 60^\circ} = \frac{87}{\sqrt{3}} = 29\sqrt{3}$$

Again in the right angled $\triangle DAC$, we have $\tan 30^\circ = \frac{DC}{AC}$

Thus, $$AC = \frac{87}{\tan 30^\circ} = 87\sqrt{3}$$

Therefore, the distance covered by the balloon is

$$ED = BC = AC - AB$$
$$= 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3} \text{ m}.$$

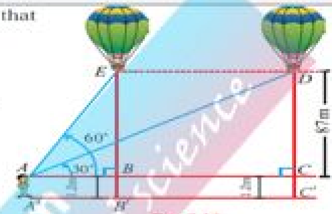


Fig. 7.16

A flag post stands on the top of a building. From a point on the ground, the angles of elevation of the top and bottom of the flag post are 60° and 45° respectively. If the height of the flag post is 10m, find the height of the building. (use $\sqrt{3} = 1.732$)

Solution

Let A be the point of observation and B be the foot of the building.

Let BC denote the height of the building and CD denote height of the flag post.

Given that $\angle CAB = 45^\circ$, $\angle DAB = 60^\circ$ and $CD = 10$ m

Let $BC = h$ metres and $AB = x$ metres.

Now, in the right angled $\triangle CAB$,

$$\tan 45^\circ = \frac{BC}{AB}.$$

Thus, $AB = BC$ i.e., $x = h$ (1)

Also, in the right angled $\triangle DAB$,

$$\tan 60^\circ = \frac{BD}{AB}$$

$$\Longrightarrow AB = \frac{h+10}{\tan 60^\circ} \quad \Longrightarrow x = \frac{h+10}{\sqrt{3}} \qquad (2)$$

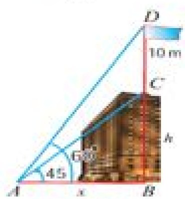


Fig. 7.17

From (1) and (2), we get $h = \frac{h+10}{\sqrt{3}}$

$$\Longrightarrow \sqrt{3}h - h = 10$$

$$\Longrightarrow h = \left(\frac{10}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \frac{10(\sqrt{3}+1)}{3-1}$$

$$= 5(2.732) = 13.66 \text{ m}$$

Hence, the height of the building is 13.66 m.

A man on the deck of a ship, 14 m above the water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the height of the cliff.

Solution Let BD be the height of the cliff.

Let A be the position of ship and E be the point of observation so that $AE = 14$ m.

Draw EC parallel to AB such that $AB = EC$.

Given that $\angle ABE = 30^\circ$, $\angle DEC = 60^\circ$

In the right angled $\triangle ABE$, $\tan 30^\circ = \frac{AE}{AB}$

Fig. 7.18

$\therefore \quad AB = \frac{AE}{\tan 30^\circ} \implies AB = 14\sqrt{3}$ Thus, $EC = 14\sqrt{3}$ ($\because AB = EC$)

In the right angled $\triangle DEC$, $\tan 60^\circ = \frac{CD}{EC}$

$\therefore \quad CD = EC \tan 60^\circ \implies CD = (14\sqrt{3})\sqrt{3} = 42$ m

Thus, the height of the cliff, $BD = BC + CD = 14 + 42 = 56$ m.

The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds horizontally, the angle of elevation changes to 30°. If the aeroplane is flying at a speed of $200\,\text{m/s}$, then find the constant height at which the aeroplane is flying.

Solution Let A be the point of observation.

Let E and D be positions of the aeroplane initially and after 15 seconds respectively.

Let BE and CD denote the constant height at which the aeroplane is flying.

Given that $\angle DAC = 30^\circ$, $\angle EAB = 60^\circ$.

Let $BE = CD = h$ metres.

Let $AB = x$ metres.

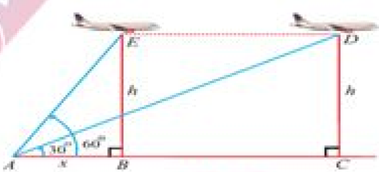


Fig. 7.19

The distance covered in 15 seconds,

$ED = 200 \times 15 = 3000$ m (distance travelled = speed $\times$ time)

Thus, $BC = 3000$ m.

In the right angled $\triangle DAC$, $\tan 30^\circ = \frac{CD}{AC}$ $CD = AC \tan 30^\circ$

Thus, $h = (x + 3000)\frac{1}{\sqrt{3}}$. (1)

In the right angled $\triangle EAB$, $\tan 60^\circ = \frac{BE}{AB}$ $BE = AB \tan 60^\circ \implies h = \sqrt{3}\,x$ (2)

From (1) and (2), we have $\sqrt{3}\,x = \frac{1}{\sqrt{3}}(x + 3000) \implies 3x = x + 3000 \implies x = 1500$ m.

Thus, from (2) it follows that $h = 1500\sqrt{3}$ m.

The constant height at which the aeroplane is flying, is $1500\sqrt{3}$ m.